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Bouncing universe and non-BPS branes

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Abstract: We describe string frame bouncing universe scenarios involving the creation and annihilation of a non-BPS D9-brane in type IIA superstring theory. We find several classes of solutions, in which the bounce is driven by the tachyon dynamics of the non-BPS brane. The metric and the dilaton are consistently described in terms of the lowest order effective action. The bounce solutions interpolate between contracting and expanding pre-big bang (or post-big bang) solutions. The singular behavior of our bounce solutions is the same as that of the pre-big bang (or post-big bang) solution. Upon adding a simple dilaton potential the asymptotic curvature singularity is removed but the dilaton still grows without bound. Such a potential may result from α' corrections in the open string sector.

Keywords: string theory and cosmology, cosmic singularity.

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1. Introduction

The resolution of the big-bang singularity is an important open problem in standard cosmology (see e. g. [1, 2] for a review and references therein). At the same time it is a natural playground for string theory since quantum gravity corrections are expected to be relevant in this regime. In the ekpyrotic scenario [3, 4] and a refined version, the cyclic universe [5], the hot big bang is the result of the collision of two branes. Explicit cyclic models have been suggested as effective four-dimensional models inspired from heterotic M-theory. The bounce in the ekpyrotic scenario occurs at a real curvature singularity and thus does not resolve the curvature singularity. The new ekpyrotic scenario [6], realizes an explicit bounce dynamics by addition of a ghost condensate but suffers from a vacuum instability problem (see [7]). More generally, due to the necessary violation of the null energy condition (NEC) during a bounce, phenomenological models producing a bounce often suffer from the problem of introducing matter with negative energy density, i. e. ghosts.

There are other ways to address the big-bang singularity problem in string theory[1]. The pre-big bang scenario (see [11] for a review) is a consequence of the fact that the tree-level equations of motion of string theory are not only symmetric under time reflection $t \mapsto -t$ but also symmetric under the scale-factor duality transformation $a \mapsto \frac{1}{a}$ with an appropriate transformation of the dilaton. The 'post-big bang' solution of standard cosmology with decelerated expansion defined for positive times is by these dualities connected to an inflationary 'pre-big bang' solution for negative times. In this way the cosmic evolution is

[1] Alternative suggestions avoiding the problem of introducing matter with negative energy density include loop quantum cosmology [8] and matrix models (e. g. [9, 10]).

extended to times prior to the big bang in a self-dual way but the solution is still singular. One can obtain regular self-dual solutions by tuning a suitable potential for the dilaton. But albeit a potential of this form might be the result of higher-loop quantum corrections, its form has not been derived from string theory.

In this paper we consider a novel scenario in string theory where a bounce occurs in the string frame due to the creation of an unstable (non-BPS) brane as the universe bounces through a string size regime before expanding as the brane decays. Our solutions effectively interpolate either between a contracting and an expanding pre-big bang solution or between a contracting and an expanding post-big bang solution. The future (past) singularity of the pre (post)-big bang solution is not resolved in this scenario. The nice feature in our model is that the curvature as well as the dilaton and its derivative remain small (in string units) through the bounce so that referring to perturbative string theory and the simplest low energy effective action for these fields is justified. In addition no fine tuning is required. On the other hand, we do not address issues like dilaton and moduli stabilization which are of course important problems to embed this model into late time standard cosmology but are not relevant during the string scale regime where the bounce occurs. We should stress that our string frame bounce solutions describe monotonously contracting or expanding geometries in the Einstein frame. A bounce in the Einstein frame may occur upon stabilizing the dilaton asymptotically. However, this entails violating the NEC or, alternatively, allowing the gravitational coupling to change sign in the string frame. We will discuss a model where the latter effect occurs.

Our work is organized as follows. This paper consists of seven sections, of which this introduction is the first. The model we employ is described in Section 2. In this section we present the effective actions and the equations of motions for the metric, dilaton and tachyon from a non-BPS brane, which makes the bounce possible. In Section 3 we show that our model has no ghost by considering the null energy condition in the Einstein frame, and some features of string frame bounce scenarios are studied in relation to the Einstein frame. In Section 4, the asymptotic behavior of the solutions is analyzed and its qualitative similarity to pre-big bang scenario is clarified. The numerical determination of the global bounce solution is presented in Section 5. In Section 6 we present a simple model that resolves the asymptotic curvature singularity in the string frame. Finally we conclude in the last section.

2. The model

We consider a non-BPS space-filling D9-brane in type IIA superstring theory. The details of the compactification will not play a role here. Concretely we consider the lowest order effective action for the metric and dilaton in the string frame as well as an effective action for the open tachyonic mode of the non-BPS D-brane. For now, the only assumption being made for the tachyon action is that only the first derivatives of the tachyon appear in it. The ansatz for the gravitational action is justified provided the dilaton and metric are slowly

varying in string units. We write

$$S = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g}\, \mathrm{e}^{-2\Phi} \left(R + 4\, \partial_\mu \Phi \partial^\mu \Phi \right) + S_{\mathrm{T}} \tag{2.1}$$

$$\text{with} \quad S_{\mathrm{T}} = \int \mathrm{d}^{10}x \sqrt{-g}\, \mathrm{e}^{-\Phi} L(T, \partial_\mu T \partial^\mu T), \tag{2.2}$$

where Φ is the dilaton, T is the tachyon, and $\kappa_{10}^2 = 8\pi G_{10}$ with G_{10} the ten-dimensional Newton constant. We use the signature $(-, +, \ldots, +)$ for the metric. With these conventions, the matter energy-momentum tensor is given by

$$T^\mu_\nu = \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{T}}(T, \partial_\rho T \partial^\rho T, \Phi)}{\delta g^\nu_\mu}. \tag{2.3}$$

For a Lagrangian minimally coupled to gravity, the metric appears only in $(\partial T)^2 = \partial_\rho T \partial^\rho T$, and we can write the energy-momentum tensor as

$$T^\mu_\nu = 2\, \mathrm{e}^{-\Phi} \frac{\partial L(T, (\partial T)^2)}{\partial \left((\partial T)^2 \right)} \partial^\mu T \partial_\nu T - \delta^\mu_\nu\, \mathrm{e}^{-\Phi} L \tag{2.4}$$

In a homogeneous isotropic universe that we will consider, all fields are assumed to depend only on time; the energy density ϵ and pressure p are given by

$$\epsilon = T^0_0 \quad , \qquad p = -T^i_i \text{ (no sum) } = \mathrm{e}^{-\Phi} L. \tag{2.5}$$

We are now ready to make an ansatz for the metric. We take a four-dimensional spatially flat FRW spacetime times a six-torus characterized by a single modulus σ. Namely

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\, \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j + \mathrm{e}^{2\sigma(t)}\, \delta_{IJ} \mathrm{d}x^I \mathrm{d}x^J, \tag{2.6}$$

where the lower-case Latin indices run over the three uncompactified space coordinates, while the upper-case Latin indices label the six compactified dimensions. We further simplify the problem by restricting to the case where $a(t) = \mathrm{e}^{\sigma(t)}$. Although equality of the two scale factors is phenomenologically not satisfying at late times it may be assumed near the cosmological bounce which is the prime focus of this paper. In particular, we will not address the important problem of moduli- and dilaton stabilization required to connect to the standard cosmology at late times. With the ansatz (2.6) the Einstein equations are now effectively isotropic. And in particular the relations (2.5) can be used. From now on the indices i, j include I, J.

The equations of motion for g^0_0, g^i_i and Φ are then given by the following first three equations

$$72H^2 - 36H\dot{\Phi} + 4\dot{\Phi}^2 - 2\kappa_{10}^2\, \mathrm{e}^{2\Phi}\, \epsilon = 0, \tag{2.7}$$

$$2\ddot{\Phi} - 8\dot{H} + 16H\dot{\Phi} - 2\dot{\Phi}^2 - 36H^2 - \kappa_{10}^2\, \mathrm{e}^{2\Phi}\, p = 0, \tag{2.8}$$

$$2\ddot{\Phi} + 18H\dot{\Phi} - 2\dot{\Phi}^2 - 9\dot{H} - 45H^2 - \frac{\kappa_{10}^2}{2}\, \mathrm{e}^{2\Phi}\, p = 0, \tag{2.9}$$

$$\dot{\epsilon} + 9H(\epsilon + p) - \dot{\Phi}\, p = 0, \tag{2.10}$$

and the last equation follows from the generalized conservation law $\nabla_\mu T^\mu_\nu = (\partial_\nu \Phi)\,\mathrm{e}^{-\Phi}\,L$.

In the case where $\epsilon = 0$ and $p = 0$, these equations allow exact solutions, namely $H = \pm\frac{1}{3t}$, $\dot{\Phi} = \frac{-1\pm 3}{2t}$. These are the special cases of the pre -big bang ($t < 0$) and post-big bang ($t > 0$) solutions, which respect the time reflection symmetry ($t \mapsto -t$) and scale-factor duality symmetry ($a \mapsto a^{-1}$) (see for example [12]).

Let us explore the possibility of the bounce. Subtracting Eq. (2.9) from Eq. (2.8) we find

$$\dot{H} + 9H^2 - 2H\dot{\Phi} - \frac{\kappa_{10}^2}{2}\,\mathrm{e}^{2\Phi}\,p = 0\,. \tag{2.11}$$

This is an important equation. It tells us that a necessary condition to have a bounce,

$$H = 0 \qquad \text{and} \qquad \dot{H} > 0\,, \tag{2.12}$$

is that the tachyon pressure p must be *positive*. This is a tight constraint for a scalar field action; the Born-Infeld action for instance, which is an often used ansatz as a higher derivative scalar field action, gives a pressure that is always negative.

Furthermore, assuming that H is negative during a contracting phase with growing dilaton and a negative pressure, (2.11) implies $\dot{H} < 0$, i. e. accelerated contraction. On the other hand, for positive equation of state for the scalar field, $w > 0$, the conservation equation in (2.10) gives a growing pressure p in the contracting phase so that a “turn around” $\dot{H} = 0$ is compatible with (2.11).

Of course, a Born-Infeld action for the tachyonic sector of non-BPS branes is not in any way suggested by string theory. On the other hand, within the restriction to first derivative actions it is possible to derive an approximate effective action from string theory for the open string tachyon of an unstable brane. This action, constructed in [13] and further studied in [14] is given by

$$L = -\sqrt{2}\,\tau_9\,\mathrm{e}^{-\frac{T^2}{2\alpha'}}\left(\mathrm{e}^{-(\partial T)^2} + \sqrt{\pi(\partial T)^2}\;\mathrm{erf}\left(\sqrt{(\partial T)^2}\right)\right), \tag{2.13}$$

where τ_9 is the tension of a BPS 9-brane, and therefore $\sqrt{2}\,\tau_9$ is the tension of a non-BPS 9-brane [15]. Let us shortly summarize how this action was constructed. First, setting $(\partial T)^2$ to zero, we see that the potential is given by

$$V(T) = \sqrt{2}\,\tau_9\,\mathrm{e}^{-\frac{T^2}{2\alpha'}}\,. \tag{2.14}$$

This is the exact potential for the open string tachyon potential found in boundary superstring field theory [16, 17, 18]. The locations of the minima of $V(T)$ are at $T = \pm\infty$. At these values the energy is degenerate with the closed string vacuum which means that the non-BPS brane is absent. The construction of the full action (2.13) is based on the observation that the tachyon kink $T(x) = \chi\,\sin(x/\sqrt{2\alpha'})$, where x is one of the spatial world volume coordinates, is an exactly marginal deformation of the underlying boundary conformal field theory [19, 20] and thus should be a solution of the equations of motion obtained from (2.13). It turns out that this requirement determines uniquely the action

once the potential has been chosen. Furthermore, it follows by analytic continuation that Sen's rolling tachyon solution [21]

$$T(t) = A\,\sinh(t/\sqrt{2\alpha'}) + B\,\cosh(t/\sqrt{2\alpha'})$$

is also a solution of the action (2.13) for all values of A and B. In this dynamical decay (or creation) of the non-BPS brane the energy is conserved. The asymptotic state for large positive (or negative) times has been argued to be given by "tachyon matter" - essentially cold dust made from very massive closed string states.

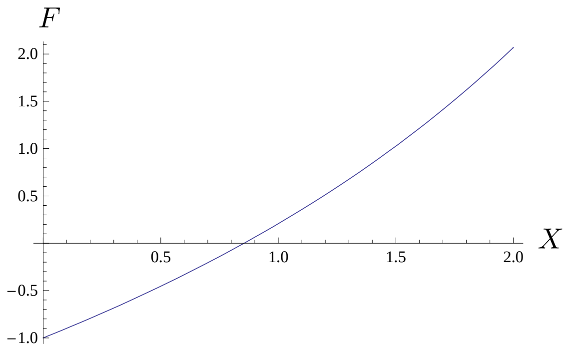


Figure 1: X-dependence of the pressure $F(X)$.

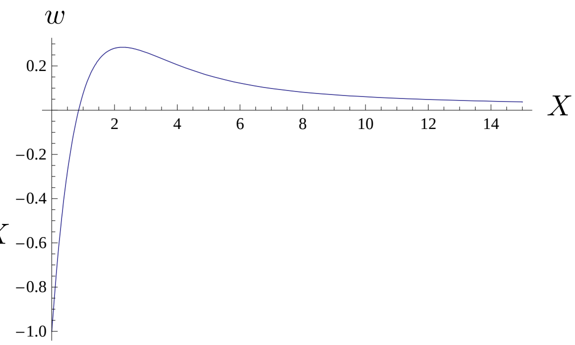


Figure 2: Equation of state w of the tachyon as a function of X.

Let us now briefly explain how this action can allow a positive pressure, or equivalently a positive Lagrangian [14]. This follows from the fact that it is real and continuous also for negative values of $(\partial T)^2$. Indeed if we write $-(\partial T)^2 = X$ (note that $X = \dot{T}^2$ in the homogeneous case), we can see that

$$\sqrt{-\pi X}\,\operatorname{erf}(\sqrt{-X}) = -2\sqrt{X}\int_0^{\sqrt{X}} \mathrm{e}^{s^2}\,\mathrm{d}s\,. \tag{2.15}$$

This is negative and grows in absolute value faster than the first term e^X in the Lagrangian; so for positive enough X (for negative enough $(\partial T)^2$), the Lagrangian, and thus the pressure, is always positive. This can be seen from Fig. 1, where we show the X - dependence of the pressure, $F(X) \equiv -\left(\mathrm{e}^X - 2\sqrt{X}\int_0^{\sqrt{X}} \mathrm{e}^{s^2}\,\mathrm{d}s\right)$. In terms of F the Lagrangian can be expressed as $L = \sqrt{2}\tau_9 \mathrm{e}^{-\frac{T^2}{2\alpha'}} F(X)$. From the figure it is clear that $\mathrm{d}F/\mathrm{d}X > 0$ (this is also clear from $\frac{\mathrm{d}F}{\mathrm{d}X} = \frac{1}{\sqrt{X}}\int_0^{\sqrt{X}} \mathrm{e}^{s^2}\,\mathrm{d}s > 0$). From now on we work in the unit system with $\alpha' = 1/2$. The energy density and pressure of the tachyon are

$$\epsilon = \sqrt{2}\tau_9 \mathrm{e}^{-\Phi}\mathrm{e}^{-T^2+\dot{T}^2} \quad , \qquad p = \sqrt{2}\tau_9 \mathrm{e}^{-\Phi}\mathrm{e}^{-T^2} F(\dot{T}^2)\,. \tag{2.16}$$

Note that the equation of state $w = \epsilon/p$ depends only on $\dot{T}^2$ as shown in Fig. 2. In particular, $w \to 0$ as $\dot{T}^2 \to \infty$, while $w \to -1$ as $\dot{T}^2 \to 0$.

3. Einstein frame and null energy condition

In this section we show that our model has no ghost and satisfies NEC in the Einstein frame. This is important because there might be a pathology due to an instability coming

from NEC violation. On the other hand, the consideration on NEC will help us draw more general conclusions concerning the bounce scenarios.

The action (2.1)-(2.2) in the string frame with L given in (2.13) is expressed in the Einstein frame by means of a conformal transformation $g_{\mu\nu} = \tilde{g}_{\mu\nu}\, \mathrm{e}^{\frac{\Phi}{2}}$ (where $\tilde{g}_{\mu\nu}$ is the metric in the Einstein frame). This yields

$$S = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-\tilde{g}} \left(\tilde{R} - \frac{(\tilde{\nabla}\Phi)^2}{2} \right) + S_{\mathrm{T}} \tag{3.1}$$

$$\text{with} \quad S_{\mathrm{T}} = \int \mathrm{d}^{10}x \sqrt{-\tilde{g}}\, \mathrm{e}^{\frac{3\Phi}{2}} L(T, \tilde{X}\mathrm{e}^{-\frac{\Phi}{2}}) \,, \tag{3.2}$$

where $\tilde{R}$ and $\tilde{\nabla}$ are the scalar curvature and the covariant derivative associated with $\tilde{g}$, and $\tilde{X} = -(\tilde{\nabla}T)^2 = X\mathrm{e}^{\frac{\Phi}{2}}$.

The relevant quantity for verifying classical stability and nonexistence of ghost is the sign of the slope of the kinetic term with respect to the first derivative of the fields, which should be positive. If it is negative, it signals the existence of a ghost (quantum mechanical vacuum instability) and also that the squared speed of sound is negative (classical instability) [7]. Since the dilaton has the correct sign for the kinetic term in the Einstein frame, the only possible source of ghost is the tachyon Lagrangian. Therefore, one should only check the sign of $\frac{\mathrm{d}L}{\mathrm{d}\tilde{X}}$. We have that $\frac{\mathrm{d}L}{\mathrm{d}\tilde{X}} = \frac{\mathrm{d}L}{\mathrm{d}X}\frac{\mathrm{d}X}{\mathrm{d}\tilde{X}} \sim \mathrm{e}^{\frac{\Phi}{2}}\frac{\mathrm{d}F}{\mathrm{d}X} = \frac{\mathrm{e}^{\frac{\Phi}{2}}}{\sqrt{X}}\int_0^{\sqrt{X}} \mathrm{e}^{s^2}\,\mathrm{d}s > 0$. So there is no ghost and therefore no violation of NEC in our model.

Now we turn to the issue of the bounce. In the spatially flat FRW spacetime we are considering, the Hubble parameter H_{E} in the Einstein frame is related to that in the string frame by $H_{\mathrm{E}} = H - \dot{\Phi}/4$. Since the null energy condition is not violated in our model, the Hubble parameter in the Einstein frame monotonously decreases, so that the bounce cannot arise in the Einstein frame. If the dilaton were frozen, it would be impossible to have a bounce in the string frame as well because the two frames would be trivially related (in particular $H_{\mathrm{E}} = H$). This is why the running dilaton is crucial for the string frame bounce. Now we make some remarks on the bounce scenario in the string frame. We assume that the bounce arises as an interpolation between two out of four different phases (i. e. contracting or expanding pre/post-big bang phases) in the pre-big bang scenario. Four transitions are then possible, namely from the contracting pre-big bang phase to expanding pre/post-big bang, or from contracting post-big bang to expanding post/pre-big bang. But the transition from pre-big bang to post-big bang cannot happen in our model because the NEC in the corresponding Einstein frame is not violated. To show this, we note that in the pre-big bang scenario with $S_{\mathrm{T}} = 0$ the solutions are

$$H = \frac{n}{t - t_0} \quad , \qquad \dot{\Phi} = \frac{9n - 1}{2(t - t_0)} \quad \text{with} \quad n = \pm\frac{1}{3}\,. \tag{3.3}$$

Here, $t < t_0$ corresponds to the pre-big bang phase, and $t > t_0$ to the post-big bang phase. It then follows that $H_{\mathrm{E}} = \frac{1-n}{8(t-t_0)}$ is negative for pre-big bang solutions and positive for post-big bang solutions, meaning that pre/post-big bang solutions correspond to a contracting/expanding universe in the Einstein frame, regardless of whether the universe

is contracting or expanding in the string frame. This means that a transition from pre-big bang to post-big bang in the string frame corresponds to a bounce in the Einstein frame, which is impossible unless the NEC is violated. Therefore, under the assumption mentioned above, the only possible string frame bounce scenarios in our model are the interpolations either between two pre-big bang phases or between two post-big bang phases.

This argument can be extended to the case where the asymptotic behavior of the solutions in the string frame is qualitatively, but not exactly, in agreement with the pre-big bang scenario, This is the case in our model, as we will see in the next section. In conclusion, if one is given a model that does not violate the NEC and if one knows the asymptotic boundary conditions of the solution in the string frame, one can then predict the possible bouncing scenarios in the string frame by looking at the corresponding Einstein frame. An example will be given in the next section.

4. Asymptotic analysis

In this section we will try to obtain approximate analytic solutions. This will provide us with the asymptotic boundary conditions for the numerical solutions in the next section. We emphasize here that by "asymptotic" we mean $t \to -\infty$, or t approaching a pole t_0, at which H diverges, as in the pre-big bang scenario. Similarly the present analysis applies to $t \to \infty$, or t approaching a pole t_0, at which H diverges as in a post-big bang scenario

To simplify our analysis, we will assume that in either of these limits, the tachyon behaves like dust, i.e. $p = w(t)\epsilon$ with $w(t) \to 0$. This is equivalent to claiming that $|\dot{T}| \to \infty$ asymptotically because $p \propto \epsilon/(\dot{T})^2$ when $|\dot{T}| \to \infty$ (see [14] for details). To justify this assumption, we look at the tachyon equation of motion following from (2.13). We have

$$\ddot{T} + \left(9H - \dot{\Phi}\right) D(\dot{T}) - T = 0\,, \tag{4.1}$$

where the function $D(y) = \mathrm{e}^{-y^2} \int_0^y \mathrm{e}^{s^2} \mathrm{d}s$ is known as the *Dawson integral*. This function is an odd function, and thus vanishes at $y = 0$. We will also use the fact that $D(y) = \frac{1}{2y} + \mathcal{O}(y^{-3})$ for $|y| \to \infty$. For $|t| \to \infty$, we will see that $(9H - \dot{\Phi})$ tends to zero. We can thus ignore the second term in the equation of motion[2] (4.1). Thus $\ddot{T} = T$ and T, as well as $\dot{T}$, will grow exponentially; and the pressure will thus vanish for $|t| \to \infty$. We emphasize that since the pressure vanishes *exponentially fast*, we can simply remove it from the asymptotic equations of motion because it will always be dominated by the other terms that will vanish with a power law.

When t approaches a pole t_0 the analysis is slightly different because there the term $(9H - \dot{\Phi})$ diverges (unless $\dot{\Phi} \sim 9H$, but we will see that this does not happen on our numerical solution). We will further assume that this term diverges at least as fast as $\frac{1}{t-t_0}$. Assuming that $\dot{T}$ is finite at t_0 this then implies that either T or $\ddot{T}$ will diverge like $\frac{1}{t-t_0}$ or faster, in contradiction with a finite $\dot{T}$. We cannot exclude the case where $\dot{T}(t_0)$ vanishes in such a way that it precisely cancels the divergence in $(9H - \dot{\Phi})$. In that case the second

[2] Note that we ignore the possibility $T \to 0$ in the infinite past or future; we are only interested in the cases where the D-brane is absent in these limits.

term of the equation of motion could be regular at t_0, and thus T and $\ddot{T}$ could be regular there as well. We will nevertheless ignore this possibility because $\dot{T}(t_0)$ corresponds to very particular initial conditions. For generic initial conditions we will therefore have that $|\dot{T}| \to \infty$ when $t \to t_0$. This then justifies our claim that we can ignore the pressure for the asymptotic analysis. An immediate consequence of Eq. (2.10) is then that $\epsilon \sim a^{-9}$.

We now will proceed by analyzing the system of equations (2.7-2.10) assuming that either $\dot{\Phi} \propto H$, $|\dot{\Phi}| \ll |H|$ or $|\dot{\Phi}| \gg |H|$ asymptotically.

i) Let us first consider the possibility $\dot{\Phi} \propto H$. In that case (2.8) and (2.9) imply that either $|\dot{H}| \gg H^2$ or $\dot{H} \propto H^2$. In the first case we get $\dot{\Phi} \simeq 5H$ and then (2.7) implies that

$$-8H^2 = 2\kappa_{10}^2 \mathrm{e}^{2\Phi} \epsilon \tag{4.2}$$

i.e. negative energy. We thus exclude that possibility. In the second case from (2.8-2.9) we obtain the solutions of the pre-big bang scenario, i. e. Eq. (3.3). For consistency, we must verify that these solutions satisfy the constraint (2.7). This is the case only when the energy density is subdominant compared to the other terms in this equation. By using (3.3), one can see that $\mathrm{e}^{2\Phi}\epsilon$ goes like $\frac{1}{|t-t_0|}$ while the other terms behave like $\frac{1}{(t-t_0)^2}$, so the energy is subdominant as $t \to t_0$. Thus the solutions can be approximated to those of the pre-big bang scenario near the pole, t_0. At the same time we see that this possibility is excluded for $|t| \to \infty$.

ii) For $|\dot{\Phi}| \ll |H|$ Eqs. (2.8) and (2.9) imply $27H^2 + 5\dot{H} = 0$. On the other hand, setting $p = 0$ in Eq. (2.11) gives us $\dot{H} + 9H^2 = 0$, a clear contradiction. Thus, $|\dot{\Phi}| \ll |H|$ is excluded.

iii) We are thus left with the sole possibility $|\dot{\Phi}| \gg |H|$. In that case (2.7) implies

$$2\dot{\Phi}^2 = \kappa_{10}^2 \mathrm{e}^{2\Phi} \epsilon, \tag{4.3}$$

and (2.8) together with (2.9) imply $-\ddot{\Phi} + \dot{\Phi}^2 = 0$, which gives $\Phi = -\log(|t - t_0|)$. With $p = 0$, Eq. (2.11) then implies $\dot{H} - 2H\dot{\Phi} = 0$, which gives $H = \frac{h}{(t-t_0)^2}$, where h is some constant. This is consistent with Eq. (4.3) only for $|t| \to \infty$ because $\mathrm{e}^{2\Phi}\epsilon \sim \frac{1}{t^2}$ as $|t| \to \infty$.

To summarize, we find that the only consistent asymptotic solution for $|t| \to \infty$ is given by

$$\Phi \simeq -\log(|t|)\ , \qquad\qquad H \simeq \frac{h}{t^2}. \tag{4.4}$$

and (3.3) for $t \to t_0$. Note that $H_{\mathrm{E}} \simeq -\frac{\dot{\Phi}}{4}$ for (4.4).

Now we are in the position to predict the possible string frame bounce scenarios. Following the same logic as in the last part of the previous section concerning the NEC in the Einstein frame, one can show that the only possible bounce scenario is the transition either from pre-big bang-like solution[3] to pre-big bang solution or from post-big bang solution to

[3]Note that what we refer to as pre/post-big bang-like solution is given in Eq. (4.4) with negative/positive time

post-big bang-like solution. This is because the other transitions correspond to a bounce in Einstein frame, which is excluded in our model which satisfies the NEC. To explain this in more detail, we consider the case where we start with contracting pre-big bang-like phase. For large negative times our bounce solution is in agreement with the pre-big bang-like solution with accelerated contraction of the universe and growing dilaton. Then the universe goes through a bounce and for $t \to t_0$ it approaches a pre-big bang solution with accelerated expansion and growing dilaton. The Hubble parameter in the Einstein frame remains negative and keeps decreasing. As will be seen in the next section, the numerical solutions are in good agreement with this picture.

5. Numerical results

In this section, we numerically solve Eqs. (2.7-2.10) to obtain global solutions. In what follows we set $2\sqrt{2}\kappa_{10}^2\tau_9 = 1$ (this can always be achieved by adding a suitable constant to the dilaton), so that

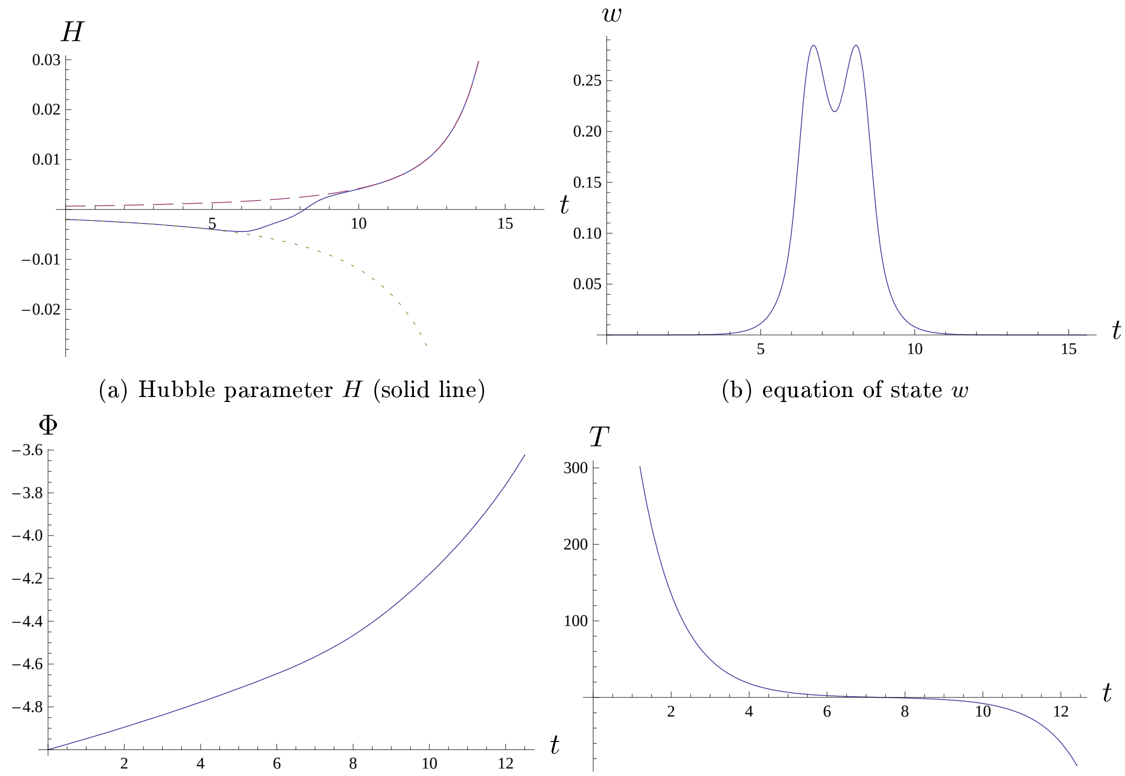
$$2\kappa_{10}^2\epsilon = \mathrm{e}^{-\Phi}\mathrm{e}^{-T^2+\dot{T}^2} \tag{5.1}$$

$$2\kappa_{10}^2\, p = -\mathrm{e}^{-\Phi}\mathrm{e}^{-T^2}\left(\mathrm{e}^{\dot{T}^2} - 2\sqrt{\dot{T}^2}\int_0^{\sqrt{\dot{T}^2}} \mathrm{e}^{s^2}ds\right). \tag{5.2}$$

With this setup, we performed the numerical analysis and found a family of bounce solutions. For example, Figs. 3 show a bounce solution with the initial conditions, $\Phi(0) = -5$, $\dot{\Phi}(0) = 0.05$, $T(0) = 1000$ and $H(0) = -0.002$. The graph in Fig. 3a shows the evolution of the Hubble parameter (solid line). The bounce takes place near $t = 8$. The bouncing solution can be seen as a transition from the contracting pre-big bang-like phase (short dashed line) to the expanding one (long dashed line). Both asymptotic solutions are obtained by setting $p = 0$ in the equations of motions since the pressure is negligible in the far future and past (see Fig. 3b). The 'double bump' feature of the equation of state can be understood by noting that as the non-BPS brane builds up $|\dot{T}|$ decreases and thus w increases from zero as explained in section 2. Then as T reaches the top of the potential $|\dot{T}|$ becomes small and consequently w decreases again. Indeed for $\dot{T} = 0$ the equation of state is that of a cosmological constant.

We found that a broad range of the initial conditions are allowed for the bounce, so there is no fine-tuning problem. For instance, $T(0) = \pm 10^4$ and $T(0) = \pm 100$ (keeping the same initial conditions as above for the other variables) gives bounce solutions with essentially the same behavior. Note that the large initial values for T do not represent a fine tuning. Rather it reflects the condition that the non-BPS brane is absent at very early times. We see that the asymptotic behavior of this family of solutions is similar to the expanding pre-big bang case, in which the Hubble parameter blows up. This agrees with the results of the previous section.

If one changes the sign of $\dot{\Phi}(0)$, a very different kind of bounce is obtained. For instance, Figs. 4 show a bounce solution with the initial conditions, $\Phi(0) = -5$, $\dot{\Phi}(0) = -0.15$, $T(0) = 5$ and $H(0) = -0.005$. The graph in Fig. 4 shows the evolution of the Hubble parameter H. The bounce takes place near $t = 2$, and the universe smoothly evolves to the



(a) Hubble parameter H (solid line)

(b) equation of state w

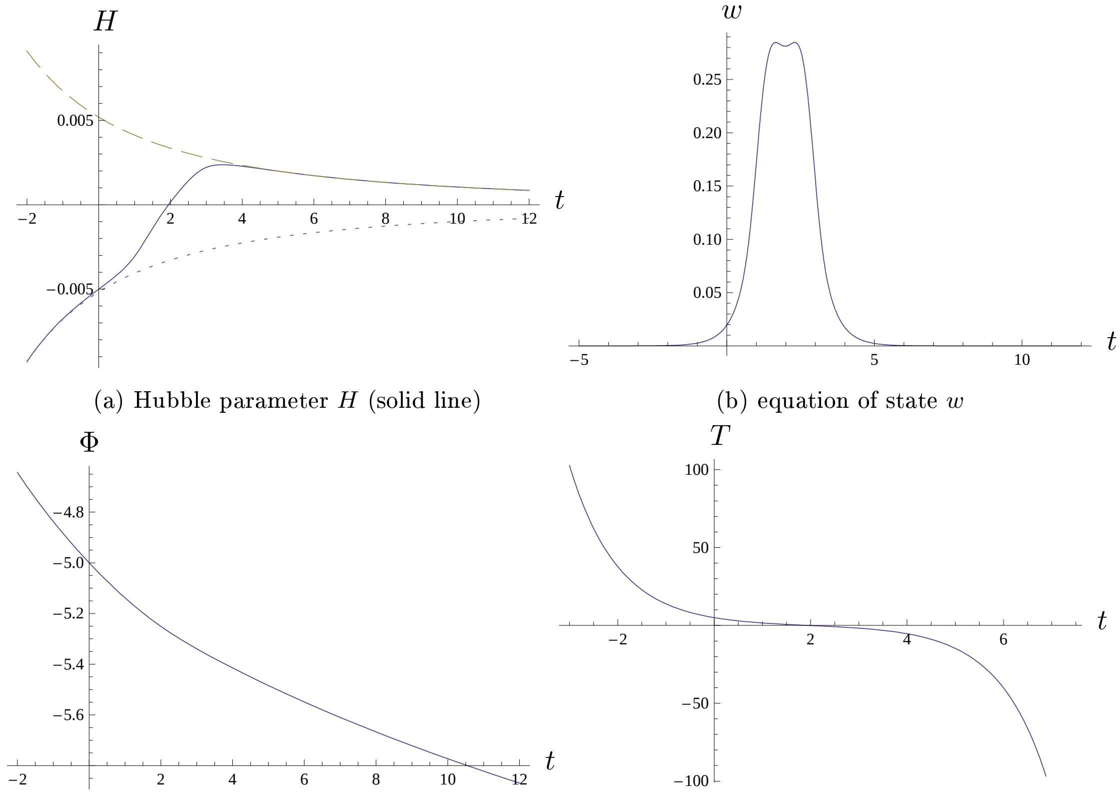
(c) dilaton Φ

(d) tachyon T

Figure 3: Bouncing numerical solution with the initial conditions $\Phi(0) = -5$, $\dot{\Phi}(0) = 0.05$, $T(0) = 1000$ and $H(0) = -0.002$.

standard cosmological regime, where the Hubble parameter and the dilaton decreases with time (Fig. 4). But going back in time further, we found a singularity where the Hubble parameter blows up. This solution can be seen as a transition from the contracting post-big bang phase (short dashed line) to the expanding post-big bang-like phase (long dashed line). Both asymptotic solutions are obtained by setting $p = 0$ in the equations of motion since the pressure is negligible in the far future and past (see Fig. 4).

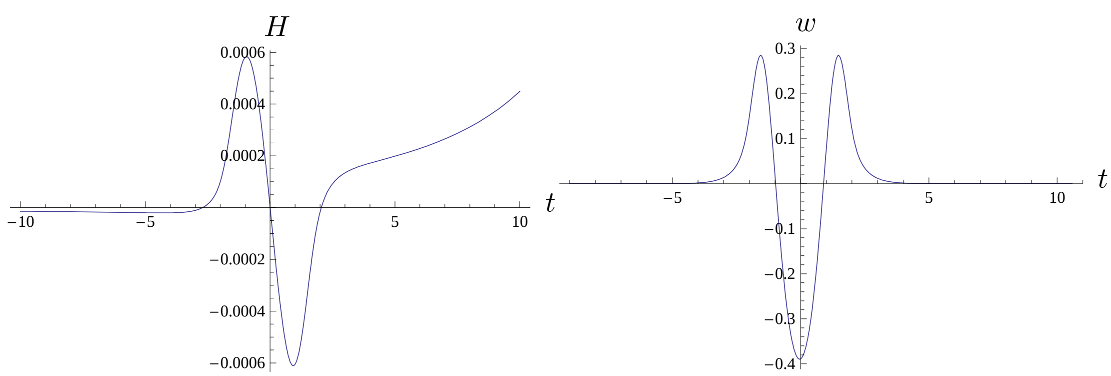
In addition, we found oscillatory solutions, in which a double bounce takes place (see Figs. 5). The solution in Figs. 6 can be seen as a time reflected one of the solution of Figs. 5. Which solution is obtained depends on the sign of $\dot{\Phi}(0)$. In both cases the asymptotic behaviors towards the curvature singularity are analogous to the non-oscillatory cases mentioned above. The evolutions of the equation of state are shown in Figs. 5 and 6. Note that the negative equation of state implies small $|\dot{T}|$ (see Fig. 2), and this means that the oscillatory solutions can arise if the speed of the tachyon is small around the top of the tachyon potential (i. e. near $T = 0$). Alternatively this kind of solution can be obtained when we arrange $|\dot{\Phi}|$ to be small enough around the top of the tachyon potential (see Fig. 7). The variation of $\dot{\Phi}(0)$ with the other initial conditions fixed (in this case at $\Phi(0) = -5$, $H(0) = 0$, and $T(0) = 0$) shows that decreasing $\dot{\Phi}(0)$ gives rise to a transition from single bounce to cyclic bounce.



(a) Hubble parameter H (solid line) (b) equation of state w

(c) dilaton Φ (d) tachyon T

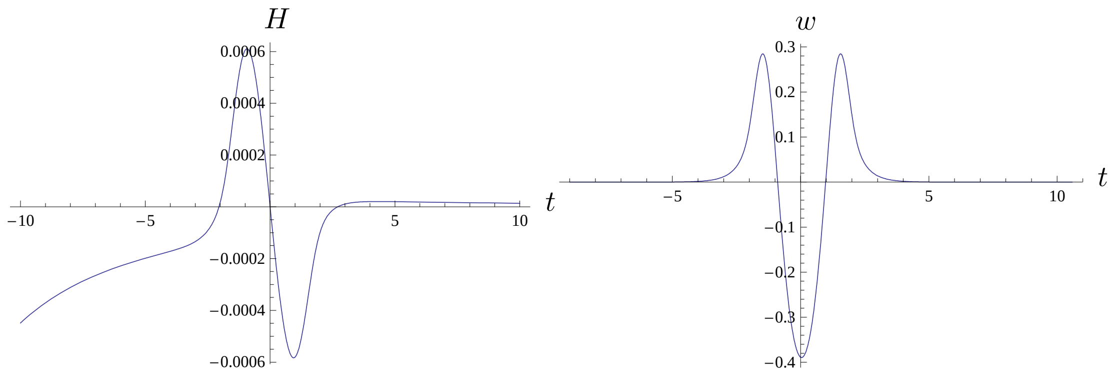
Figure 4: Bouncing, numerical solution with non-singular future using initial conditions $\Phi(0) = -5$, $\dot{\Phi}(0) = -0.15$, $T(0) = 5$ and $H(0) = -0.005$.



(a) Hubble parameter H (b) equation of state w

Figure 5: Oscillatory, numerical solution with initial conditions $\Phi(0) = -5$, $H(0) = 0$, $\dot{\Phi}(0) = 0.05$ and $T(0) = 0$.

In conclusion, we obtained bounce solutions, where the universe smoothly evolves from the contracting phase to the expansion. The bounce scenario that we here found can be classified into the following two cases. Note that these are exactly in agreement with our predictions on the bounce scenarios in the previous section:



(a) Hubble parameter H (b) equation of state w

Figure 6: Oscillatory, numerical solution with initial conditions $\Phi(0) = -5$, $H(0) = 0$, $\dot{\Phi}(0) = -0.05$ and $T(0) = 0$.

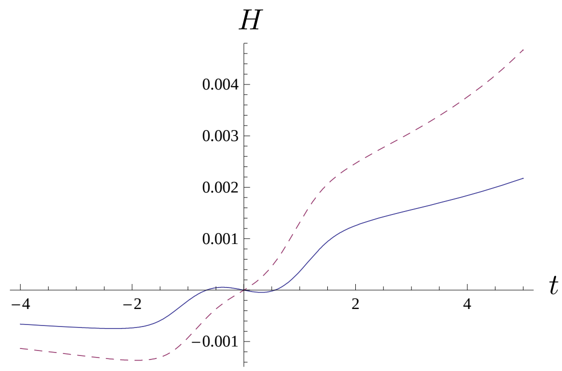


Figure 7: Hubble parameter H for $\dot{\Phi}(0) = 0.06$ (solid line) and $\dot{\Phi}(0) = 0.072$ (dashed line).

1. Transition from accelerating contraction (the contracting pre-big bang-like phase) to accelerating expansion (the pre-big bang inflation): In this case the dilaton grows up, and if the speed of the tachyon (or dilaton) is small enough near the maximum of the tachyon potential, a double bounce can take place (Figs. 5) before the universe evolves to the pre-big bang phase.

2. Transition from decelerating contraction (the contracting post-big bang phase) to decelerating expansion (post-big bang like phase): In this case the dilaton decays, and a double bounce can also take place (Figs. 6) under the same condition mentioned above.

In all cases the tachyon rolls over the top of the potential in the course of its evolution, and the bounce seems to happen when the tachyon reaches around the top of the potential. The pressure is important only around the bounce, and negligible (dust) asymptotically.

Our string frame bounce solutions correspond to monotonously contracting (i. e. $H_{\mathrm{E}} = H - \frac{\dot{\Phi}}{4} < 0$) or expanding geometries ($H_{\mathrm{E}} > 0$) in the Einstein frame (see Figs. 8-9), meaning that there is no bounce in the Einstein frame for our solutions. This is the fact that our model does not violate the NEC in the Einstein frame.

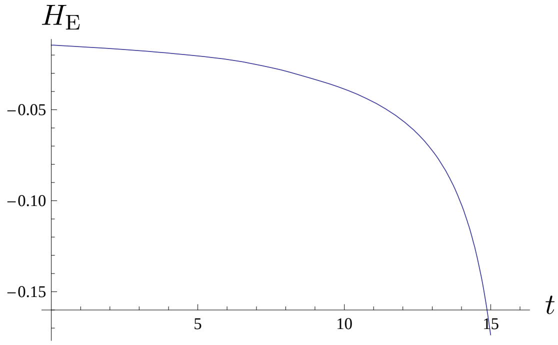


Figure 8: Hubble parameter H_E in the Einstein frame corresponding to Fig. 3 (a).

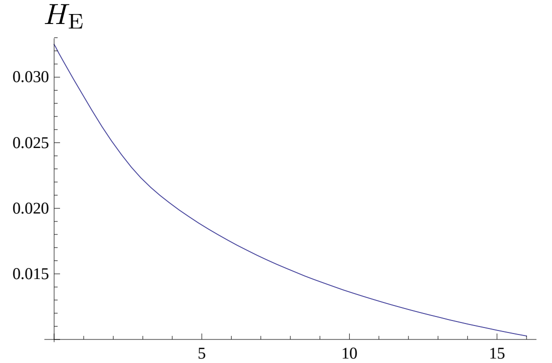


Figure 9: Hubble parameter H_E in the Einstein frame corresponding to Fig. 4 (a).

The numerical results presented in this section verify the results on the asymptotics in the previous section. Interestingly, as far as the dilaton Φ is concerned the asymptotic solution obtained here agrees qualitatively with the global numerical solution. In other words the dynamics of the dilaton is not much affected by the presence of the non-BPS brane and is qualitatively similar to that of the pre-big bang scenario. Concerning $H(t)$ things are different: See Fig. 3 for example. For large negative times $H(t)$ is well described by the asymptotic solution described in Eq. (4.4). Then near the bounce which takes place at $t \simeq 0$ [4] the non-BPS brane affects $H(t)$ significantly. Then for $t \to t_0 > 0$ (near the pole), where the pre big-bang singularity occurs, the Hubble constant is well described by the solution in the pre-big bang scenario. This can be understood from the fact that the brane has already decayed for $t \to t_0$.

6. Nonsingular solutions – Example

As we have seen in the previous section, there is a singularity either in the future or in the past, depending on the sign of $\dot{\Phi}$, and we expect that this singularity may be resolved in the same way as in pre-big bang scenarios, e. g. relying on α' corrections or quantum loop corrections or alternatively using a dilaton potential (see [12] for some explicit examples). As a matter of fact, resolving this kind of asymptotic singularities is less difficult than the big-bang singularity. In this section we will give an example of resolution of this asymptotic singularity.

Near the singularity the curvature and the dilaton blow up, and this suggests that a potential term of the form $R\,\mathrm{e}^{\Phi}$ in the Lagrangian may smooth out the singularity. Here the coupling between the Ricci scalar and the dilaton is introduced because the singularity appears both in the curvature and dilaton. Such a term is quite likely to appear as α' correction in the open string sector, since it has the form of a tree level correction in the open string coupling constant and R is the natural invariant built from background metric derivatives[5].

[4] In fact one can always arrange the bounce to take place at $t = 0$ by shifting the time variable.

[5] A similar potential has been motivated in the context of string gas cosmology in [22] as a Casimir-type potential.

Thus as an example, in which such an additional term may resolve the singularities, we study the dynamics of the system where the action (2.1) is supplemented by a potential

$$\frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}\mathrm{e}^{-2\Phi}RV(\Phi). \tag{6.1}$$

The equations of motion then take the form

$$72H^2\left(1+V(\Phi)\right)+4\dot{\Phi}^2-36H\dot{\Phi}\left(1+V(\Phi)-\frac{V'(\Phi)}{2}\right)-2\kappa_{10}^2\mathrm{e}^{2\Phi}\epsilon=0 \tag{6.2}$$

$$2\ddot{\Phi}-2\dot{\Phi}^2+18H\dot{\Phi}-\left(9\dot{H}+45H^2\right)\left(1+V(\Phi)-\frac{V'(\Phi)}{2}\right)-\kappa_{10}^2\mathrm{e}^{2\Phi}p/2=0 \tag{6.3}$$

$$\begin{aligned}(\ddot{\Phi}+8H\dot{\Phi})\left(V'(\Phi)-2V(\Phi)-2\right)+\dot{\Phi}^2\left(V''(\Phi)-4V'(\Phi)+4V(\Phi)+2\right)+\\ \left(1+V(\Phi)\right)(36H^2+8\dot{H})+\kappa_{10}^2\mathrm{e}^{2\Phi}p=0\end{aligned} \tag{6.4}$$

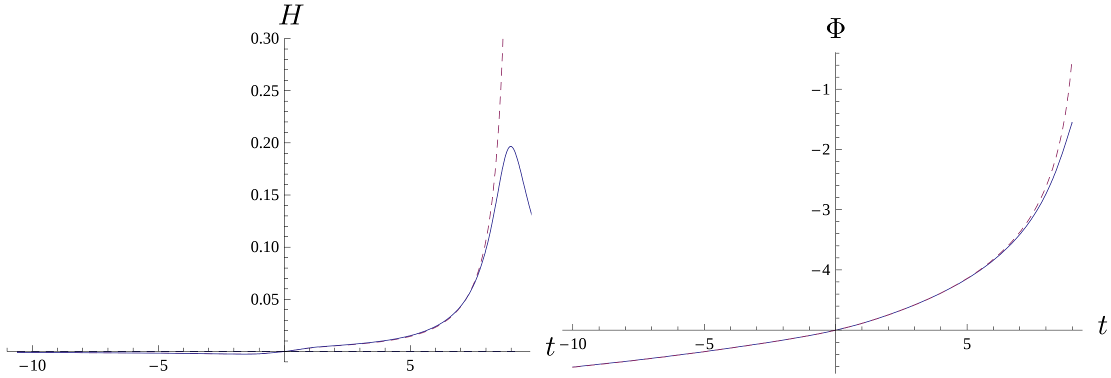
$$\dot{\epsilon}+9H(\epsilon+p)-\dot{\Phi}p=0. \tag{6.5}$$

We require that the additional term should not spoil the bounce, namely this term is important only when the curvature becomes very big. For concreteness we choose $V=-\mathrm{e}^{\Phi+5}/40$. Using the same setup as in the previous section, we perform the numerical analysis.

First, let us consider the case in which the dilaton grows; in this case we faced a future singularity. We found that the addition of a potential (6.1) can resolve the future curvature singularity. This is shown in Figs. 10, where the dashed (solid) curve corresponds to the case without (with) the additional term. Here the initial conditions are chosen such that the bounce takes place at $t=0$ (in other words we impose the initial conditions at the bounce and extrapolate in both directions in time). In the case without the additional term, there is a singularity, while in the other case the universe evolves to the standard cosmological regime where the Hubble parameter decreases. As can be seen from the plot, the dynamics are almost the same in both cases before the Hubble parameter gets significantly big, so that the bounce is not spoiled. Once the Hubble parameter is large enough, the additional term smooths out the singularity.

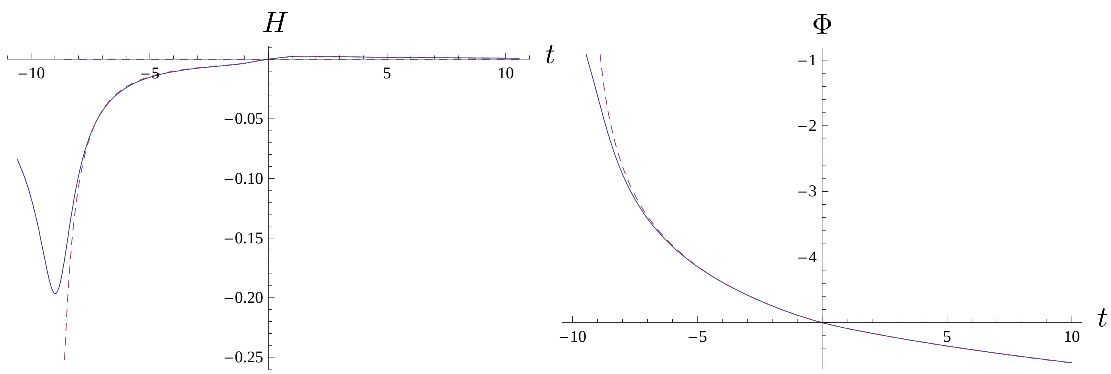
Now we turn to the case in which the dilaton decreases. In the previous section we have seen that in this case there is a past singularity. With the help of the additional term mentioned above, we found that this singularity can be resolved as well. This is shown in Figs. 11. The mechanism of resolving the singularity is analogous to the case of growing dilaton that we have seen above. What is interesting is that this solution corresponds to the time reflected version of the case of growing dilaton since $H\mapsto -H$ and $\dot{\Phi}\mapsto-\dot{\Phi}$ under the time reflection, $t\mapsto -t$.

In both cases, the dilaton dynamics still has a singularity either in the future or in the past. This is in contrast to the case where the tachyon sector is absent (i. e. $\epsilon=0$ and $p=0$). (see Figs. 12, where the pre-big bang singularity is resolved not only in H, but also in Φ). Since the singular behavior of our solutions is the same as in the pre-big bang scenario, we expect that in principle the dilaton singularity can be resolved as in Figs. 12, but this may require fine-tuning.



(a) Hubble parameter H (b) dilaton Φ

Figure 10: Numerical solution with growing dilaton (solid lines: with additional term, dashed lines: without additional term).



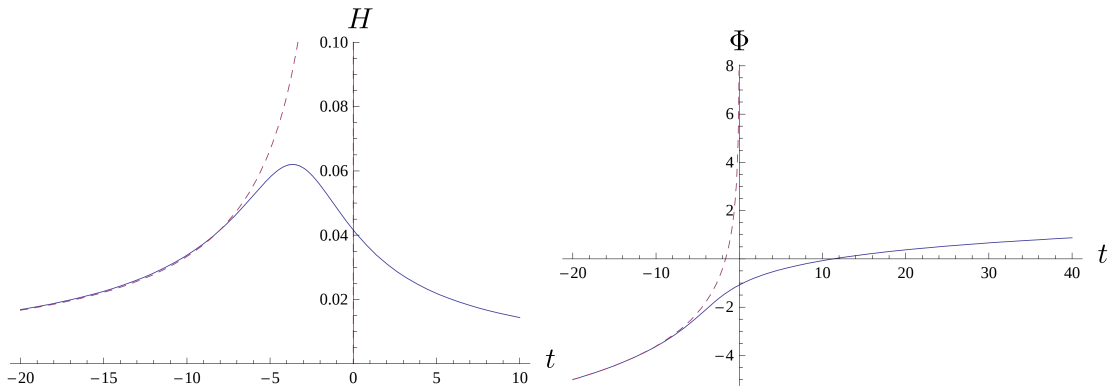
(a) Hubble parameter H (b) dilaton Φ

Figure 11: Numerical solution with decreasing dilaton (solid lines: with additional term, dashed lines: without additional term).

To sum up, we have shown an explicit example in which an additional term that might arise from higher order corrections can resolve the curvature singularity without affecting the bounce dynamics, though the singularity in dilaton has not been resolved.

7. Conclusion

We suggested bounce scenarios, in which a non-BPS space-filling D9-brane in type IIA superstring theory drives a bounce of the scale factor in the string frame. We employed the lowest order effective action for the metric and dilaton in the string frame as well as an effective action for the open tachyonic mode of the non-BPS D-brane. The positivity of the pressure of the tachyon field is responsible for the bounce, which is why the DBI action, for instance, can not drive the bounce. The curvature as well as the time derivative of the dilaton remain small during the bounce. In other words, the gravitational sector is entirely classical.



(a) Hubble parameter H (b) dilaton Φ

Figure 12: Numerical solution with tachyon sector absent (solid lines: with the potential given in (6.1), dashed lines: pre-big bang solution).

Asymptotically our bounce solutions look like pre-big bang or post-big bang solutions, with singular behavior of the curvature and the dilaton. The asymptotic string frame curvature singularity can be resolved by adding a phenomenological potential, $\propto R\mathrm{e}^{-\Phi}$, which may or may not result from α' corrections in the open string sector. It would be desirable to determine the sign and the precise numerical value of the proportionality coefficient. With our choice of the sign the gravitational coupling changes sign in the string frame. This results in a bounce in the Einstein frame at some time after the bounce has taken place in the string frame without violating the null energy condition. An interesting observation is that while our phenomenological potential stabilizes the dilaton within the perturbative regime it fails to do so once the tachyonic sector is included. An obvious question is then whether a modified potential exists which stabilizes the dilaton in our model, and if so, whether it can be derived from string theory. We should also mention that throughout this paper we assumed the isotropy in 9-dimensional space (modulo compactification, the details of which did not play a role here) during the era of the bounce. For phenomenological reasons it may be preferable to consider scenarios with a different dynamics for the scale factor of the internal dimensions. In particular, orbifold compactifications are interesting since they are accompanied by a reduction of supersymmetry. A preliminary analysis shows that for a T^6/Z_2 orbifold the asymptotic solutions in the far past are not modified qualitatively. The numerical evolution of the global solution requires more work, however. We hope to report on this issue in a future publication.

Acknowledgments

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